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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 26 February, 2021 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. If $30.{}^{30}C_0 + 29.{}^{30}C_1 + \ldots\ldots + 1.{}^{30}C_{29} = n.2^m$ where m is a natural number and greatest common divisor of n, 2 is 1, then the value of (n + m) is

(1) 45 (2) 50 (3) 60 (4) 75

Ans. **(1)**

Sol. $(1 + x)^{30} = {}^{30}C_0 + {}^{30}C_1\, x + {}^{30}C_2\, x^2 + {}^{30}C_3 x^3 + \ldots\ldots\ldots + {}^{30}C_{30} x^{30}$

Differentiate w.r.t x

$\Rightarrow 30(1 + x)^{29} = {}^{30}C_1 + {}^{30}C_2 .\, 2x + \ldots\ldots + 30.{}^{30}C_{30}\, x^{29}$

Put x = 1

$\Rightarrow 30 \times 2^{29} = {}^{30}C_1 + {}^{30}C_2 .\, 2 + \ldots\ldots.. + 30.{}^{30}C_{30}$

$\Rightarrow 15 \times 2^{30} = 30.{}^{30}C_0 + 29.{}^{30}C_1 + \ldots\ldots + 1.\, {}^{30}C_{29}$

so n = 15
m = 30 because gcd (15,2) = 1
so (n + m) = 15 + 30 = 45

2. Consider a cubic equation $x^3 - 2x^2 + 2x - 1 = 0$, then the sum of 162^{th} powers of its roots is
(1) 3 (2) 5 (3) 7 (4) 6

Ans. **(3)**

Sol. $x^3 - 2x^2 + 2x - 1 = 0$
$(x-1)(x^2 - x + 1) = 0$
$x = 1,\ x = -\omega,\ x = -\omega^2$
$1^{162} + (-\omega)^{162} + (-\omega^2)^{162}$
$1 + \omega^{162} + \omega^{324}$
$1 + 1 + 1 = 3$

3. The value of $\int_0^{\pi} |\sin 2x| dx$ is

Ans. **(2)**

Sol. $\int_0^{\pi} |\sin 2x| dx = 2\int_0^{\frac{\pi}{2}} \sin 2x\, dx$ (using property $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$, where $f(2a - x) = f(x)$)
$= (-\cos 2x)_0^{\pi/2} = 2$

4. The number of 7 digit numbers which are formed by using the digits 1, 2, 3 only and the sum of its digits is 10, is
(1) 77 (2) 35 (3) 42 (4) 82

Ans. **(1)**

Sol. There are two possibilities
Case-I
$1, 1, 1, 1, 1, 2, 3 = \frac{7!}{5!} = 42$
Case -II
$1, 1, 1, 1, 2, 2, 2 = \frac{7!}{4!3!} = 35$
Total $42 + 35 = 77$

5. A is symmetric square matrix of second order with integer entries such that trace of A^2 is 1, then the number of such matrices A is -
(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(4)**

Sol. $\begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}$
$= \begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{bmatrix}$
$a^2 + 2b^2 + c^2 = 1$
$\Rightarrow$ case - I
$a = \pm 1,\ b = 0,\ c = 0$
case -II
$a = 0,\ b = 0,\ c = \pm 1$
total four matrices

6. If a series $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$.(upto infinite terms) is given, then the value of S is
(1) $\frac{13}{4}$ (2) $\frac{15}{4}$ (3) $\frac{17}{4}$ (4) 1

Ans. **(1)**

Sol. $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$ ....(i)
$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \ldots$ ....(ii)
$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \ldots$

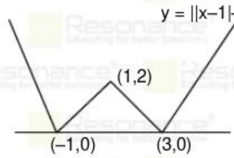
$$\frac{2S}{3} = \frac{4}{3} + 5\left(\frac{\frac{1}{3^2}}{1-\frac{1}{3}}\right)$$

$$\frac{2S}{3} = \frac{4}{3} + 5\left(\frac{\frac{1}{3^2}}{\frac{2}{3}}\right) = \frac{13}{4}$$

7. Area bounded by curve y=||x–1|–2| and x axis is

Ans. (4)

Sol.



$\Delta = \frac{1}{2} \times 4 \times 2 = 4$ sq. unit

8. $\int_{-\pi/2}^{\pi/2}\left(\frac{\cos^2 x}{1+3^x}\right)dx$

(1) $\pi/4$ (2) 2π (3) 4π (4) $\pi/2$

Ans. (1)

Sol. $I = \int_{-\pi/2}^{\pi/2}\left(\frac{\cos^2 x}{1+3^x}\right)dx$ (using property $\int_{-a}^{a} f(x)dx = \int_{0}^{a}(f(x)+f(-x))dx$)

$I = \int_{0}^{\pi/2} \cos^2 x\,dx$

$I = \int_{0}^{\pi/2}\left(\frac{1+\cos 2x}{2}\right)dx = \frac{\pi}{4}$

9. If 3sinx + 4cosx = k + 1, then the number of possible integral values of k is

(1) 9 (2) 10 (3) 11 (4) 12

Ans. (3)

Sol. $-5 \le 3\sin x + 4\cos x \le 5$

$-5 \le k+1 \le 5$

$-6 \le k \le 4$

k = –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4

$\Rightarrow$ 11 integral values of k

10. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$ is (where [•] denotes greatest integer function)

Ans. (1) 100(e+1) s(2) 100(e–1) (3) 99 (e–1) (4) 100(1–e)

Ans. (2)

Sol. $= \int_0^{100} e^{\{x\}}dx = 100\int_0^1 e^{\{x\}}dx = 100\int_0^1 e^x dx = 100(e-1)$ Ans.

11. A fair coin is tossed a fixed number of times and probability of getting 7 heads is equal to probability of getting 9 heads. Then the probability of getting 2 heads.

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{16}}$ (3) $\frac{15}{2^{12}}$ (4) None of these

Ans. (1)

Sol. Let coin is tossed n times

$^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$

$^nC_7\left(\frac{1}{2}\right)^n = {}^nC_9\left(\frac{1}{2}\right)^n \Rightarrow {}^nC_7 = {}^nC_9 \quad \Rightarrow n = 16$

$\therefore$ required probability = ${}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14} = {}^{16}C_2\left(\frac{1}{2}\right)^{16}$

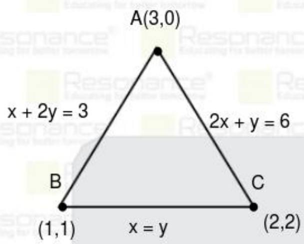
$= \frac{15}{2^{13}}$

12. A triangle is formed using three lines $x + 2y = 3$, $x = y$, $2x + y = 6$. Then this triangle is.

(1) equilateral (2) Isosceles (3) Right angle triangle (4) None of these

Ans. **(2)**

Sol.



$AB = \sqrt{5}$; $BC = \sqrt{2}$; $AC = \sqrt{5}$

AB = AC

$\Rightarrow$ Triangle is isosceles

13. In expansion $\left(t\,x^{1/5} + \frac{(1-x)^{1/10}}{t}\right)^{10}$ $\forall\, x \in (0,1)$, then the greatest value of term which is independent of t is

(1) $\frac{10!}{3(5!)^2}$ (2) $\frac{10!}{\sqrt{3}(5!)^2}$ (3) $\frac{2}{3}\frac{10!}{(5!)^2}$ (4) $\frac{2}{3\sqrt{3}}\frac{10!}{(5!)^2}$

Ans. **(4)**

Sol. $T_{r+1} = {}^{10}C_r\,(tx^{1/5})^{10-r}\left[\frac{(1-x)^{1/10}}{t}\right]^r$

$= {}^{10}C_r\,\frac{t^{10-r}}{t^r}\left(x^{\frac{10-r}{5}}\right)(1-x)^{r/10}$

For the term Independent of t

So $10 - 2r = 0$

$\boxed{r = 5}$ $\therefore$ T_6 will be independent of t

So $T_6 = {}^{10}C_5\ \ x\,(1-x)^{1/2}$

Now let $f(x) = x(1-x)^{1/2}$

$f'(x) = 0 = (1-x)^{1/2} - \frac{1}{2}\frac{x}{\sqrt{1-x}} \Rightarrow x = \frac{2}{3}$

$\therefore$ greatest term independent to 't' $T_6 = {}^{10}C_5\left(\frac{2}{3}\right)\left(1-\frac{2}{3}\right)^{1/2}$

$T_6 = \frac{2}{3\sqrt{3}}\frac{10!}{(5!)^2}$

14. Evaluate $\lim\limits_{h\to 0}\dfrac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{h\left(\sqrt{3}\sinh - \cosh\right)}$

(1) –4 (2) $\dfrac{-4}{\sqrt{3}}$ (3) 4 (4) $\dfrac{4}{\sqrt{3}}$

Ans. (1)

sol. $\lim\limits_{h\to 0}\dfrac{2\times 2\left(\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)\right)}{h\left(\sqrt{3}\sinh-\cosh\right)}$

$= \lim\limits_{h\to 0}\dfrac{4\left(\sin\left(\frac{\pi}{6}+h\right)\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos\left(\frac{\pi}{6}+h\right)\right)}{h\left(\sqrt{3}\sinh-\cosh\right)}$

$= \lim\limits_{h\to 0}\dfrac{4\sinh}{h\left(\sqrt{3}\sinh-\cosh\right)}$

$= \lim\limits_{h\to 0}\dfrac{4}{(-1)} = -4$

15. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) 0 (2) $\frac{1}{2}|a|^4\,\vec{b}$ (3) $|a|^4\vec{b}$ (4) $\vec{b}$

Ans. (3)

sol. $\vec{a}\times\left(\vec{a}\times\vec{b}\right)$

$= (a.b)\ \vec{a} - |a|^2\ \vec{b}$

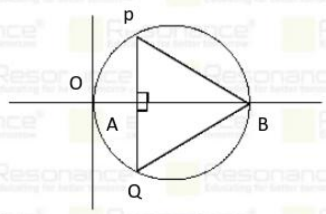
$= -|a|^2\vec{b}$

$= -\vec{a}\times(\vec{a}\times(|a|^2\,\vec{b}) = -|\vec{a}|^2(\vec{a}\times(\vec{a}\times\vec{b}))$

$= -|\vec{a}|^2(-|\vec{a}|^2\vec{b})$

$= |a|^4\vec{b}$

16. A circle circumscribing a Δ BQP and passing through origin such that OA = 1 and OB = 13. Then the the area of ΔPQB , in which OB ⊥ PQ is



(1) $24\sqrt{2}$ (2) $12\sqrt{2}$ (3) $24\sqrt{3}$ (4) $24\sqrt{3}$

Ans. (3)

Now OB = 13 , OA = 1,OB ⊥ PQ

OA.AB = AP.AQ (but AP = AQ)

$\Rightarrow$ AP $= 2\sqrt{3}$

Area of Δ PQB = AP.AB = $24\sqrt{3}$

17. If $\dfrac{\sin^{-1}x}{a}=\dfrac{\cos^{-1}x}{b}=\dfrac{\tan^{-1}y}{c}$ then the value of cos $\left(\dfrac{\pi c}{a+b}\right)$ is

(1) $\dfrac{1-y^2}{1+y^2}$ (2) $\dfrac{2y}{1+y^2}$ (3) $\dfrac{1+y^2}{1-y^2}$ (4) $\dfrac{2y}{1-y^2}$

Ans. (1)

Ans. (1)

Sol. Let $\sin^{-1} x = \theta$

$\cos^{-1} x = \frac{\pi}{2} - \theta,$

Now $\frac{\theta}{a} = \frac{\frac{\pi}{2}-\theta}{b} = \frac{\tan^{-1} y}{c}$

$\because \frac{\theta}{a} = \frac{\frac{\pi}{2}-\theta}{b}$

$\Rightarrow b\,\theta = \frac{a\pi}{2} - a\theta$

$a + b = \frac{a\pi}{2\theta}$

Now $\cos\left(\frac{\pi c}{a+b}\right) = \cos\left(\frac{\pi c}{a\pi} \times 2\theta\right)$

$= \cos\left(\frac{2c}{a}\theta\right)$

$\because \frac{\theta}{a} = \frac{\tan^{-1} y}{c} \Rightarrow \frac{c\theta}{a} = \tan^{-1} y$

$= \cos\left(2\tan^{-1} y\right) = \frac{1-y^2}{1+y^2}$

18. If $y(x) = \frac{x^4}{2} - 5x^3 + 18x^2 - 19x$, then the ordered pair (x,y) for which $\frac{dy}{dx}$ is maximum, is

(1) (2,9) (2) (1,1) (3) (2,3) (4) (2,2)

Ans. **(4)**

Sol. $y' = 2x^3 - 15x^2 + 36x - 19$

$y'' = 6x^2 - 30x + 36$

$= 6(x-2)(x-3)$

$y'' = 0 \Rightarrow x = 2, x = 3$

$\because y''' = 12x - 30$

$y'''(2) < 0 \rightarrow$ maximum

$y'''(3) > 0$

so, at $x = 2$, $\frac{dy}{dx}$ is maximum

at $x = 2 \Rightarrow y = 2$

$\therefore (x,y) = (2,2)$

19. Suppose growth of bacteria increases according to the law of exponential growth. Initially the number of bacteria was 1000 and bacteria increase by 20% after 2 hours. Then the time when number of bacteria will become 2000 is $\frac{k}{\ln\frac{6}{5}}$. Find $\left(\frac{k}{\ln 2}\right)^2$.

(1) 4 (2) 8 (3) 16 (4) 32

Ans. **(1)**

Sol. $\frac{dN}{dt} = CN$

$\int_{1000}^{1200} \frac{dN}{N} = \int_0^2 C\,dt$

$(\ln N)_{1000}^{1200} = C(t)_0^2$

$\ln\frac{1200}{1000} = 2C$

$C = \frac{1}{2}\ln\left(\frac{6}{5}\right)$

$\therefore \int_{1000}^{2000} \frac{dN}{N} = \int_0^t c\,dt$

$(\ln N)_{1000}^{2000} = c(t)$

$(\ln N)_{1000} = c(t)$

$\because c = \frac{1}{2}\ln\left(\frac{6}{5}\right)$

$\Rightarrow \ln 2 = \frac{1}{2}\ln\left(\frac{6}{5}\right)(t) \Rightarrow t = \frac{\ln 4}{\ln\left(\frac{6}{5}\right)} \Rightarrow k = \ln 4$

$\therefore \left(\frac{k}{\ln 2}\right)^2 = \left(\frac{\ln 4}{\ln 2}\right)^2 = 4$

20. The number of values of x, satisfying the equation is $\log_4 (x-1) = \log_2 (x-3)$.
(1) 0 (2) 1 (3) 3 (4) 4

Ans. **(2)**

Sol. $\frac{1}{2}\log_2 (x-1) = \log_2 (x-3)$

$\Rightarrow (x-1) = (x-3)^2 \Rightarrow x^2 - 7x + 10 = 0 \Rightarrow (x-5)(x-2) = 0$

$x = 2, 5$

at x=2, given equation is not defined

So number of solution is 1.

21. If f is real-valued differentiable function satisfying $|f(x)-f(y)| \le (x-y)^2$, x, y$\in$R & f(0) = 0, then
(1) f(x)> 0 $\forall x\in R$ (2) f(x)< 0 $\forall x\in R$ (3) f(x) = 0, $\forall x\in R$ (4) f(x) can take any value

Ans. **(4)**

Sol. $\therefore$ f is a differentiable function, then

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \Rightarrow \quad |f'(x)| = \left|\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\right|$

$|f'(x)| \le \left|\lim_{h\to 0}\frac{(x+h-x)^2}{h}\right| \Rightarrow \quad |f'(x)| \le 0$

$|f'(x)| = 0$

f (x) is a constant function

$\therefore$ f(0) = f (1)=0

22. If in an increasing G.P, the sum of second and sixth term is $\frac{25}{2}$ and the product of third and fifth term is 25, then the sum of fourth term, sixth term & eight term is

Ans. **35**

Sol. $t_2 + t_6 = \frac{25}{2} \Rightarrow ar(1+r^4) = \frac{25}{2}$ ……(1)

$t_3 \cdot t_5 = 25$

$ar^2 \cdot ar^4 = 25$

$a^2r^6 = 25$

$ar^3 = 5$ ……(2)

using equation (1) and (2)

$\frac{ar^3}{ar(1+r^4)} = \frac{2}{5}$

$5r^2 = 2+2r^4$

$2r^4 - 5r^2 + 2 = 0$

$\Rightarrow r^2 = 2, \frac{1}{2}$,

$\boxed{r^2 = 2}$ $\left[\because r^2 = \frac{1}{2} \text{rejected}\right]$ (becuase of increasing GP)

$t_4 + t_6 + t_8 = ar^3 + ar^5 + ar^7$

$= ar^3 (1 + r^2 + r^4) = 5(1+2+4) = 35$ Ans.

23. Three planes P_1, P_2, P_3, are given $3x + 15y + 21z = 9$, $2x+y-z = 4$, $2x+10y+14z = \frac{9}{5}$ respectively then which of the following is correct.
(1) P_1 is parallel to P_2 (2) P_2 is parallel to P_3
(3) P_1 is parallel to P_3 (4) P_1, P_2, P_3, are parallel to one another

Ans. **(3)**

Sol. For parallel planes

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

so plane P_1 and P_3 are parallel

24. The value of determinant A = $\begin{vmatrix} (a+1)(a+2) & (a+2) & 1 \\ (a+2)(a+3) & (a+3) & 1 \\ (a+3)(a+4) & (a+4) & 1 \end{vmatrix}$ is equal to

(1) – 2 (2) (a + 1) (a + 2) (a + 3) (3) 0 (4) (a + 2) (a + 3) (a + 4)

Ans. **(1)**

Sol. $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ (a+3)(a+4)-(a+1)(a+2) & 2 & 0 \end{vmatrix}$$

$2(a+2)x2 - \{(a+3)(a+4) - (a+1)(a+2)\}$

$4(a+2) - \{a^2 + 7a + 12 - (a^2 + 3a + 2)\}$

4a + 8 – (4a + 10)

4a + 8 – 4a – 10

= –2

25. The number of solution(s) of $\sqrt{3}\cos^2 x = (\sqrt{3}-1)\cos x + 1$, $x \in \left[0, \frac{\pi}{2}\right]$ is

Ans. **01**

Sol. $\sqrt{3}\cos^2 x - (\sqrt{3}-1)\cos x - 1 = 0$

$\sqrt{3}\cos^2 x - \sqrt{3}\cos x + \cos x - 1 = 0$

$(\sqrt{3}\cos x + 1)(\cos x - 1) = 0$

$\cos x = \frac{-1}{\sqrt{3}}, 1$

$\cos x = 1 \Rightarrow x = 0$

Number of solution is 1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