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PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

(Reproduced from memory retention)

Date : 26 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

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MATHEMATICS

1. $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{3}$

Ans. (1)

Sol. $I = \displaystyle\int_0^{\pi/2}\left(\frac{\cos^2 x}{1+3^x}+\frac{\cos^2 x}{1+3^{-x}}\right)dx = \int_0^{\pi/2}\left(\frac{\cos^2 x}{1+3^x}+\frac{3^x\cos^2 x}{1+3^x}\right)dx = \int_0^{\pi/2}\cos^2 x\,dx$

$= \displaystyle\frac{1}{2}\int_0^{\pi/2}(1+\cos 2x)\,dx = \frac{1}{2}\left(x+\frac{1}{2}\sin 2x\right)\Bigg|_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$

2. Value of $\displaystyle\lim_{x\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+x\right)-\cos\left(\frac{\pi}{6}+x\right)}{\sqrt{3}x\,(\sqrt{3}\cos x-\sin x)}\right\}$ is equal to

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{2}{3}$ (4) $\frac{4}{\sqrt{3}}$

Ans. (1)

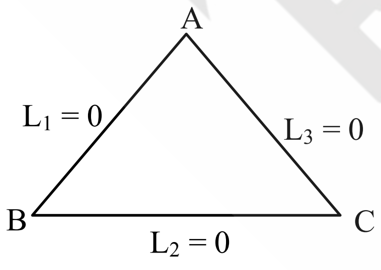
Sol. $= \displaystyle\lim_{x\to 0}\frac{2\left[\sin\left(\frac{\pi}{6}+x-\frac{\pi}{6}\right)\right]}{\sqrt{3}\,x(\sqrt{3})} = \lim_{x\to 0}\frac{4}{3}\,\frac{\sin x}{x} = \frac{4}{3}$

3. If x – y = 0, x + 2y = 3 and 2x + y = 6 are three lines forming a triangle, then the triangle is

(1) Isosceles (2) Right angled (3) Equilateral (4) None of these

Ans. (1)

Sol.



$L_1 : x - y = 0$

$L_2 : x + 2y = 3$

$L_3 : 2x + y = 6$

A (2, 2)

B (1, 1)

C (3, 0)

$\Rightarrow AB = \sqrt{2}$, $BC = \sqrt{5}$, $AC = \sqrt{5}$

$\therefore$ Triangle is isosceles

4. Find the number of integral values of k for which the equation 3 sin x + 4 cos x = k + 1 has a solution.

(1) 13 (2) 6 (3) 8 (4) 11

Ans. (4)

Sol. $-\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$

$-5 \le (k+1) \le 5$

$-6 \le k \le 4$

5. Number of 7 digits number in which sum of digits is 10 and digits can take 1, 2, 3 values, is

(1) 77 (2) 42 (3) 60 (4) 35

Ans. (1)

Sol. **Case-1:** 1, 1, 1, 1, 1, 2, 3

ways $= \frac{7!}{5!} = 42$

Case-2: 1, 1, 1, 1, 2, 2, 2

ways $= \frac{7!}{4!.3!} = 35$

total ways = 42 + 35 = 77

6. Find the number of solutions of the equation $4(x-1) = \log_2(x-3)$

(1) 0 (2) 1 (3) 2 (4) 4

Ans. (1)

Sol. $4(x-1) = \log_2(x-3)$

$2^{4(x-1)} = (x-3)$ here $x \ge 3$

So no solution

7. If A is a symmetric matrix of order 2 and sum of diagonal elements of A^2 is 1, where elements of matrix are integer, then number of such matrices are

(1) 4 (2) 6 (3) 8 (4) 5

Ans. (1)

Sol. $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$A^2 = \begin{bmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2 \end{bmatrix}$

$tr(A^2) = a^2 + 2b^2 + c^2 = 1$

$\Rightarrow$ b = 0 and $a^2 + c^2 = 1$

$\Rightarrow$ (a, c) ≡ (1, 0), (–1, 0), (0, 1), (0, –1)

8. The maximum value of slope of tangent to $y = \frac{x^4}{2} - 5x^3 + 18x^2 + 6$ is at a point.

(1) (2,2) (2) (2,46) (3) $\left(1, \frac{39}{2}\right)$ (4) (1,0)

Ans. (2)

Sol. $m = \frac{dy}{dx} = 2x^3 - 15x^2 + 36x$

$\frac{dy}{dx} = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6) = 0$

$\Rightarrow x = 2, 3$

$\frac{d^2y}{dx^2} = 6(2x - 5)$

$\left.\frac{d^2y}{dx^2}\right|_{n=2} = -ve$

$\therefore$ Maximum at x = 2

Point (2, 46)

9. {(P, Q) ; P, Q be 2 points which are equidistant from origin}, then point (x, y) which are equivalence class of (1, –1)

(1) $x^2 + y^2 = 2$ (2) $x^2 + y^2 = \sqrt{2}$ (3) $x^2 + y^2 = 1$ (4) $x^2 + y^2 = 2\sqrt{2}$

Ans. (1)

Sol. The equivalence class containing (1, –1) for this relation is $x^2 + y^2 = 2$

10. Value of $\begin{vmatrix} (a+1)(a+2) & (a+1) & 1 \\ (a+2)(a+3) & (a+2) & 1 \\ (a+3)(a+4) & (a+3) & 1 \end{vmatrix}$ is equal to

(1) –2 (2) 2 (3) 0 (4) 1

Ans. (1)

Sol. $D = \begin{vmatrix} a^2+3a+2 & a+1 & 1 \\ a^2+5a+6 & a+2 & 1 \\ a^2+7a+12 & a+3 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$D = \begin{vmatrix} a^2+3a+2 & a+1 & 1 \\ 2a+4 & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix} = 4a + 8 - 4a - 10 = -2$

11. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c}$, then find the value of $\cos\left(\frac{\pi c}{a+b}\right)$

(1) $\frac{1+y^2}{1-y^2}$ (2) $\frac{2y}{1+y^2}$ (3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{y}{1+y^2}$

Ans. (3)

Sol. Let $\sin^{-1}x = a\lambda$, $\cos^{-1}x = b\lambda$, $\tan^{-1}y = c\lambda$

$\Rightarrow (a + b)\lambda = \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{a+b}=2\lambda$

Now $\cos\left(\frac{\pi}{a+b}\right)=\cos(2\lambda c)=\cos\left(2\tan^{-1}y\right)$

$=\frac{1-y^2}{1+y^2}$

12. If $\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})))=$

(1) $|\vec{a}|^4\,\vec{b}$ (2) $-|\vec{a}|^4\,\vec{b}$ (3) $|\vec{a}|^2\,\vec{b}$ (4) $-|\vec{a}|^2\,\vec{b}$

Ans. (1)

Sol. $\vec{a}\times(\vec{a}\times((\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\vec{b}))$

$=\vec{a}\times(-|\vec{a}|^2(\vec{a}\times\vec{b}))=--|\vec{a}|^2((\vec{a}.\vec{b})\vec{a}-|\vec{a}|^2\vec{b})=-|\vec{a}|^4\,\vec{b}-|\vec{a}|^2(\vec{a}.\vec{b})\vec{a}$

$=|\vec{a}|^4\,\vec{b}$ $(\because \vec{a}.\vec{b}=0)$

13. If $|f(x)-f(y)|\le|(x-y)^2|$; x, y $\in$ R and f(0) = 1 then

(1) f(x) = 0 for x $\in$ R (2) f(x) > 0 : x $\in$ R

(3) f(x) < 0 ; x $\in$ R (4) f(x) can take any value

Ans. (2)

Sol. $\left|\frac{f(x)-f(y)}{x-y}\right|\le|x-y|$

$\Rightarrow |f'(x)| \le 0$

$\Rightarrow f'(x) = 0$

$\Rightarrow f(x) = \text{constant}$

$\Rightarrow f(x) = 1$

14. $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3}$........$\infty$

(1) $\frac{13}{4}$ (2) $\frac{13}{2}$ (3) $\frac{11}{4}$ (4) $\frac{11}{2}$

Ans. (1)

Sol. $S = 1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+......\infty$ (i)

$\frac{1}{3}S = \frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+..........$ (ii)

(i) – (ii)

$\frac{2}{3}S=1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+........$

$\frac{2}{3}S=\frac{4}{3}+\frac{5}{3^2}+\frac{5}{3^3}+..........$

$\frac{2}{3}S=\frac{4}{3}+\frac{\frac{5}{3^2}}{1-\frac{1}{3}}=\frac{4}{3}+\frac{5}{6}=\frac{13}{6}$

$S = \frac{13}{6}\times\frac{3}{2}=\frac{13}{4}$

15. Find maximum value of term independent of t in expansion of $\left(tx^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{10}$

(1) $56\sqrt{3}$ (2) $\frac{56}{\sqrt{3}}$ (3) 56 (4) $28\sqrt{3}$

Ans. (1)

Sol. $T_{r+1} = {}^{10}C_r\,(t\,x^{1/5})^{10-r}\left(\frac{(1-x)^{1/10}}{t}\right)^r$

$10 - r - r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5\, x(1-x)^{1/2}$

$\frac{d(T_6)}{dx} = {}^{10}C_5\left((1-x)^{1/2}+\frac{-x}{2\sqrt{1-x}}\right)=0$

$2(1-x) - x = 0 \Rightarrow x = \frac{2}{3}$

Maximum $T_6 = {}^{10}C_3\frac{2}{3}\left(\frac{1}{3}\right)^{1/2} = 56\sqrt{3}$

16. $\sum_{n=1}^{n=100} \int_{n-1}^{n} e^{x-[x]}\,dx$ is equal to :

(1) 100(e – 1) (2) 100 e (3) 100 (4) 100 (1 – e)

Ans. (1)

Sol. $\sum_{n=1}^{n=100} \int_{n-1}^{n} e^{\{x\}}\,dx$

$= 100 \int_{0}^{1} e^{x} = 100\,(e-1)$

17. If a fair coin is tossed n times, probability of getting 9 heads is equal to probability of getting 7 heads . Find the probability of given 2 heads.

(1) ${}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$ (2) ${}^{16}C_2 \times \left(\frac{1}{2}\right)^{14}$ (3) ${}^{16}C_3 \times \left(\frac{1}{2}\right)^{16}$ (4) ${}^{16}C_3 \times \left(\frac{1}{2}\right)^{14}$

Ans. (1)

Sol. ${}^{n}C_9 \times \left(\frac{1}{2}\right)^{9} \times \left(\frac{1}{2}\right)^{n-9} = {}^{n}C_7 \times \left(\frac{1}{2}\right)^{7} \times \left(\frac{1}{2}\right)^{n-7}$

${}^{n}C_9 = {}^{n}C_7 \Rightarrow n = 16$

$P(2\text{Heads}) = {}^{16}C_2 \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{14}$

$= {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$

18. Given three planes $P_1 : 3x - 15y + 21z = 9$

$P_2 : 4x - 20y + 21z = 10$

$P_3 : 2x - 10y + 14z = 10$

Then

(1) P_1, P_2 are parallel (2) P_1, P_2, P_3 are parallel

(3) P_1, P_3 are parallel (4) P_2, P_3 are parallel

Ans. (3)

Sol. $P_1 : x - 5y + 7z = 3$

$P_2 : 4x - 20y + 21z = 10$

$P_3 : x - 5y + 7z = 5$

P_1 and P_3 are parallel as dr's of normal are same

19. The summation of 2^{nd} & 6^{th} terms of an increasing GP is $\frac{25}{2}$ and product of 3^{rd} & 5^{th} term is 25, then summation of 4^{th}, 6^{th} & 8^{th} term is

(1) 30 (2) 35 (3) 20 (4) 22

Ans. (2)

Sol. $ar + ar^5 = \frac{25}{2}$ and $ar^2 \cdot ar^4 = 25$ $\Rightarrow ar^3 = 5$

$\therefore \frac{r+r^5}{r^3} = \frac{5}{2}$

$\Rightarrow 2 + 2r^4 = 5r^2$

$\Rightarrow 2r^4 - 5r^2 + 2 = 0$

$\Rightarrow r^2 = 2$ or

$r^2 = \frac{1}{2}$ Reject

Now, $ar^3 + ar^5 + ar^7 = 5 + ar^5(1 + r^2) = 5 + 5.2(1 + 2) = 35$

20. If P(1,5,35) Q(7,5,2) R(1,λ,7) S(2λ,1,2) are coplanar then sum of value of λ is :

(1) $\frac{39}{5}$ (2) $\frac{17}{2}$ (3) $\frac{-39}{5}$ (4) $\frac{-17}{2}$

Ans. (2)

Sol. for points to be coplanar $\begin{vmatrix} 6 & 0 & -33 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -38 \end{vmatrix} = 0$

$\Rightarrow 6(-33\lambda + 165 - 112) + 33(2\lambda^2 - 11\lambda + 5) = 0$

$\Rightarrow -198\lambda + 318 + 66\lambda^2 - 363\lambda + 165 = 0$

$\Rightarrow 66\lambda^2 - 561\lambda + 483 = 0$

Sum $= \frac{561}{66} = \frac{187}{22} = \frac{17}{2}$

21. $\int_0^{\pi} |\sin 2x|\, dx$ is equal to

Ans. 2

Sol. $\int_0^{\pi} |\sin 2x|\, dx$

Here $f(2a - x) = f(x)$

$= 2\int_0^{\pi/2} (\sin 2x)dx$

$= 2\left(-\frac{\cos 2x}{2}\right)_0^{\pi/2}$

$= 2$

22. If $30.{}^{30}C_0 + 29.{}^{30}C_1 + 28.{}^{30}C_2 + ... + {}^{30}C_{29} = n.2^m$ then find the value of (m + n)

Ans. 59

Sol. General term = $(30 - r).{}^{30}C_r$

$$\text{L.H.S} = \sum_{r=0}^{30}(30-r).{}^{30}C_r$$

$$= 30\sum_{r=0}^{30}{}^{30}C_r - \sum_{r=0}^{30}r.{}^{30}C_r$$

$= 30.2^{30} - 30.2^{29}$

$= 30.2^{29}$

So n = 30, m = 29

m + n = 59

23. If $x^3 - 2x^2 + 2x - 1 = 0$ has roots α, β, γ then find $(\alpha^{162} + \beta^{162} + \gamma^{162})$

Ans. 3

Sol. $n = 1, n = -\omega, n = -\omega^2$

$\alpha = 1, \beta = -\omega, \gamma = -\omega^2$

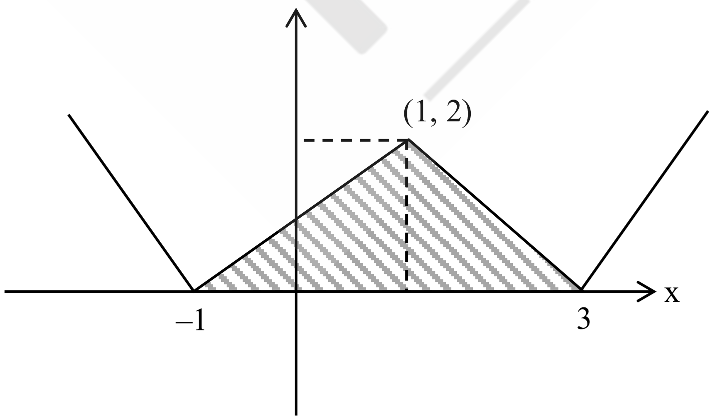
$E = 1 + \omega^{162} + (\omega^2)^{162}$

$= 3$

24. Find the area bounded by the curve $y = \left||x-1|-2\right|$ with x-aixs

Ans. 4

Sol.



$$\text{Area} = \frac{1}{2} \times 4 \times 2 = 4$$

25. Find number of solutions of $\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1$ in $x \in \left[0, \frac{\pi}{2}\right]$

Ans. 1

Sol. $\sqrt{3}\cos^2 x - (\sqrt{3} - 1)\cos x - 1 = 0$

$\cos x = \frac{(\sqrt{3}-1) \pm \sqrt{(\sqrt{3}-1)^2 + 4\sqrt{3}}}{2\sqrt{3}}$

$= \frac{(\sqrt{3}-1) \pm \sqrt{(4+2\sqrt{3})}}{2\sqrt{3}} = \frac{(\sqrt{3}-1) \pm (\sqrt{3}+1)}{2\sqrt{3}}$

$= 1, \frac{-1}{\sqrt{3}}$

since $x \in \left[0, \frac{\pi}{2}\right]$

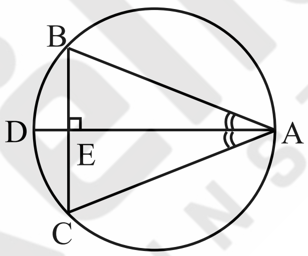
$\Rightarrow \cos x = \frac{-1}{\sqrt{3}}$, not possible

$\therefore \cos x = 1$

$\Rightarrow x = 0$

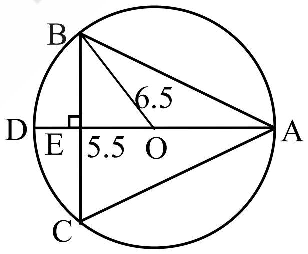
$\therefore$ number of solution 1

26. In the given figure AD = 13, DE = 1, AD bisects angle BAC and BC is perpendicular to AD, then, area of triangle ABC.



Ans. 41.568

Sol. Let O be mid-point of AD, now perpendicular from C to BC bisects chord BC, (ΔACE and ΔABE are congruent). Hence AD is diameter and O is centre of circle.



So BE = $\sqrt{(6.5)^2 - (5.5)^2}$

$= \sqrt{12}$

Hence area = $\frac{1}{2} \cdot 12.2\sqrt{12} = 24\sqrt{3}$

27. Find the difference between the value of degree and order of differential equation corresponding to the family of curves $y^2 = a\left(x+\sqrt{2}\right)$.

Ans. 2

Sol. order of differential equation is 1.

$2yy' = a$

$\Rightarrow y^2 = 2yy'\left(x+\sqrt{2yy'}\right)$

$\Rightarrow y - 2xy' = 2y'.\sqrt{2yy'}$

$\Rightarrow (y - 2xy')^2 = 4(y')^2.2yy'$

$$\Rightarrow \left(y-2x.\frac{dy}{dx}\right)^2 = 8y.\left(\frac{dy}{dx}\right)^3$$

Degree of Differential equation = 3

28. A plane is passing through (λ, 2, 1) & (4, –2, 2). It is perpendicular to line joining points A(–2, 23, 18) and B(–1, 29, 16). Find value of $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$

Ans. 8

Sol. $\overline{AB} = \hat{i}+6\hat{j}-2\hat{k}$

$\alpha = (\lambda - 4)\ \hat{i}+4\hat{j}-\hat{k}$

$\overline{AB}\ .\alpha = 0$

$\lambda - 4 + 24 + 2 = 0 \quad \Rightarrow \quad \lambda = -22$

$E = 4 + 8 - 4 = 8$

29. Number of bacteria are increasing at a rate proportional to its number at time 't' of at t = 0, N = 1000 and after 2 hours, number of bacteria increased by 20%.If at $t = \frac{k}{\ell n\frac{5}{6}}$, number of bacteria are 2000, then find $\left(\frac{k}{\ell n2}\right)^2$?

Ans. 4

Sol. $\frac{dx}{dt} \propto x$

$\Rightarrow \frac{dx}{dt} = \lambda x$

$\Rightarrow \int_{1000}^{x} \frac{dx}{x} = \lambda \int_{0}^{t} dt$

$\Rightarrow \ell n \frac{x}{1000} = \lambda t$

at t = 2, x = 1200

$\therefore\ 2\lambda = \ell n \frac{6}{5}$

$\therefore\ x = 1000.\, e^{\frac{1}{2}\ell n \frac{6}{5}.t}$

Now $2000 = 1000.\, e^{\frac{1}{2}\ell n \frac{6}{5}.\frac{k}{\ell n \frac{5}{6}}}$

$\Rightarrow 2 = e^{-\frac{k}{2}}$

$\Rightarrow \frac{k}{2} = -\ell n 2$

$\Rightarrow \frac{k}{\ell n 2} = -2$