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PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

(Reproduced from memory retention)

Date : 25 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

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MATHEMATICS

1. If $x=\sum_{n=0}^{\infty}\cos^{2n}(\theta)$, $y=\sum_{n=0}^{\infty}\sin^{2n}(\phi)$, $\theta,\phi\in\left(0,\frac{\pi}{2}\right)$ and $Z=\sum_{n=0}^{\infty}(\sin\phi)^{2n}.(\cos\theta)^{2n}$ then -

(1) $Z=\frac{xy}{xy-1}$ (2) $Z=\frac{xy}{xy+1}$ (3) $Z=\frac{xy-1}{xy}$ (4) $Z=\frac{xy+1}{xy}$

Ans. (1)

Sol. $x = 1 + \cos^2\theta + \cos^4\theta + ...= \frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta}$

$y = 1 + \sin^2\phi + \sin^4\phi + ...= \frac{1}{1-\sin^2\phi}=\frac{1}{\cos^2\phi}$

$z = 1 + \sin^2\phi\cos^2\theta + \sin^4\phi\cos^4\theta + ...= \frac{1}{1-\sin^2\phi\cos^2\theta}=\frac{1}{1-\frac{1}{x}\frac{1}{y}}$

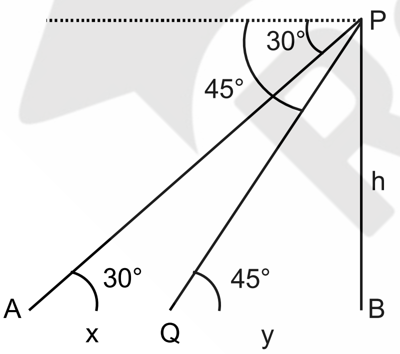
$\Rightarrow z=\frac{xy}{xy-1}$

2. A person is standing at P on a tower of height h. The angles of depression to the ground at A and Q are 30° & 45° respectively. Time taken to cover the distance AQ is 20s. Find the time to cover distance QB.

(1) $10(\sqrt{3}-1)\sec$ (2) 30 sec (3) $10(\sqrt{3}+1)\sec$ (4) $10(\sqrt{2}+1)\sec$

Ans. (3)

Sol.



In ΔABP

$\frac{h}{x+y}=\tan 30° = \frac{1}{\sqrt{3}}$

$x + y = \sqrt{3}h$ …(i)

In ΔQBP

$\frac{h}{y}=\tan 45° = 1$

$h = y$ …(ii)

$x + y = \sqrt{3}\,y$

$x = (\sqrt{3} - 1)y$

Let speed is v

$\frac{x}{v} = 20 \Rightarrow x = 20\,v$

$\therefore\ 20\,v = (\sqrt{3} - 1)\,y$

Time to cover y distance $= \frac{y}{v} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1)\,\text{sec}$

3. Image of P(3,5) on the line $y = x + 1$ is Q. Then Q lies on

(1) $(x - 4)^2 + (y - 2)^2 = 4$ (2) $(x - 1)^2 + y^2 = 4$

(3) $x^2 + y^2 = 4$ (4) $x^2 + (y - 2)^2 = 4$

Ans. (1)

Sol. Image of P(3,5) on the line $x - y + 1 = 0$ is

$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$

$x = 4, y = 4$

$\therefore$ Image is (4,4)

Which lies on

$(x - 4)^2 + (y - 2)^2 = 4$

4. If $A \to (B \to A)$ then which of the following is contrapositive

(1) $A \to (B \vee A)$ (2) $(A \wedge B)$ (3) $(B \wedge \sim A) \to A$ (4) $(B \to A)$

Ans. (1)

Sol. Contrapositive of $A \to (B \to A)$ is

$\sim (B \to A) \to \sim A$

$(B \wedge \sim A) \to \sim A$

5. Consider the parabola $y^2 = 6x$. If a tangent to the parabola is perpendicular to the line $2x + y = 1$, then which of the following point does not lie on the tangent:

(1) (5, 4) (2) (4, 5) (3) (6, 6) (4) (8, 7)

Ans. (1)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2} + 3 \Rightarrow x - 2y + 6 = 0$

6. Find the value of : $I = \int_{-1}^{1} x^2.e^{[x^3]}\,dx$, where ([.] greatest integer function)

(1) $\frac{1}{3}-\frac{1}{3e}$ (2) $\frac{1}{3}+\frac{1}{3e}$ (3) $\frac{1}{3e}-\frac{1}{2}$ (4) 2

Ans. (2)

Sol. $I = \int_{-1}^{0} x^2.e^{-1}\,dx + \int_{0}^{1} x^2\,dx$

$\therefore I = \left.\frac{x^3}{3e}\right|_{-1}^{0} + \left.\frac{x^3}{3}\right|_{0}^{1}$

$\Rightarrow I = \frac{1}{3e}+\frac{1}{3}$

7. Let $A = \begin{bmatrix} 0 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 0 \end{bmatrix}$ and $(I + A)(I - A)' = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then find the value of $13(a^2 + b^2)$

(1) $26\sec^2\frac{\theta}{2}$ (2) $13\tan^4\frac{\theta}{2}$ (3) $26\tan^2\frac{\theta}{2}$ (4) $13\sec^4\frac{\theta}{2}$

Ans. (4)

Sol. $A = \begin{bmatrix} 0 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 0 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I + A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$, $I - A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$, $(I - A)' = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\therefore\ (I + A)(I - A)' = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1-\tan^2\frac{\theta}{2} & 2\tan\frac{\theta}{2} \\ -2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$

$\therefore\ a = 1 - \tan^2\frac{\theta}{2}$, $b = -2\tan\frac{\theta}{2}$

$\therefore\ 13(a^2 + b^2) = 13\left(\left(1-\tan^2\frac{\theta}{2}\right)^2 + 4\tan^2\frac{\theta}{2}\right) = 13\left(1+\tan^2\frac{\theta}{2}\right)^2 = 13\sec^4\frac{\theta}{2}$

8. If $L_1 : (2 - i)z = 4i + (2 + i)\bar{z}$

$L_2 : (2 + i)z = (i - 2)\bar{z}$; if L_1 & L_2 intersect at a point A and a circle is drawn with centre A touching the line $(i - 1)z + (1 + i)\bar{z} + 2i = 0$, then radius of circle is -

(1) $\frac{3}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $3\sqrt{2}$ (4) $\frac{1}{3\sqrt{2}}$

Ans. (4)

Sol. $L_1 = (2 - i)(x + iy) = 4i + (2 + i)(x - iy)$

$\Rightarrow (2x + y) + i(2y - x) = (4 + x - 2y)i + (2x + y)$

$\Rightarrow 2y - x = 4 + x - 2y$

$\Rightarrow 4y - 2x = 4$

$\Rightarrow x - 2y + 2 = 0$

$L_2 = (2 + i)(x + iy) = (i - 2)(x - iy)$

$\Rightarrow 2x - y + i(2y + x) = (-2x + y) + i(x + 2y)$

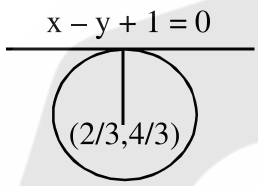
$\Rightarrow 2x - y = -2x + y$

$\Rightarrow 2x - y = 0 \quad \Rightarrow x - 2(2x) + 2 = 0$

$\Rightarrow -3x + 2 = 0 \quad \Rightarrow x = \frac{2}{3}, y = \frac{4}{3}$

$(i - 1)(x + iy) + (i + 1)(x - iy) + 2i = 0$

$-x - y + x + y = 0, x - y + x - y + 2 = 0$



$$r = \left|\frac{\frac{2}{3} - \frac{4}{3} + 1}{\sqrt{2}}\right| = \left|\frac{1}{3\sqrt{2}}\right|$$

9. $\int \frac{\sin\theta \sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1 - \cos 2\theta} d\theta$ is equal to

(1) $\frac{1}{6}[2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

(2) $\frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

(3) $\frac{1}{18}[2\cos^6\theta - 9\cos^5\theta + 18\cos^2\theta + 11]^{3/2} + C$

(4) $\frac{1}{18}[-2\cos^6\theta - 9\cos^5\theta - 18\cos^3\theta + 11]^{3/2} + C$

Ans. (2)

Sol. $\int \frac{2\sin^2\theta\cos 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{2\sin^2\theta}\,d\theta$

Let $\sin\theta = t$ $\cos\theta\, d\theta = dt$

$= \int (t^6+t^4+t^2)\sqrt{2t^4+3t^2+6}\ dt = \int (t^5+t^3+t)\sqrt{2t^6+3t^4+6t^2}\ dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)\, dt = dz$

$= \frac{1}{12}\int \sqrt{z}\ dz = \frac{1}{18} z^{3/2} + c$

$= \frac{1}{18}(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta)^{3/2} + C$

$= \frac{1}{18}[(1-\cos^2\theta)(2(1-\cos^2\theta) + 3 - 3\cos^2\theta + 6)]^{3/2} + C$

$= \frac{1}{18}[(1-\cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + C$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

10. If $\vec{a} = \hat{i}+\hat{j}-\hat{k}$, $\vec{b} = \hat{i}+\hat{j}$, $\vec{c} = \hat{i}-2\hat{j}-\hat{k}$ & $\vec{r}\times\vec{a} = \vec{c}\times\vec{a}$, $\vec{r}\,.\,\vec{b} = 0$, then $\vec{r}\,.\,\vec{a}$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $-\frac{3}{2}$ (4) $-\frac{1}{2}$

Ans. (2)

Sol. $\vec{r}\times\vec{a} = \vec{c}\times\vec{a}$

$(\vec{r}-\vec{c})\times\vec{a} = \vec{0}$

$\Rightarrow\ \vec{r}-\vec{c} = \lambda\vec{a}$

$\vec{r} = \lambda\vec{a}+\vec{c}$

$\vec{r}\,.\,\vec{b} = 0 \ \Rightarrow\ (\vec{a}\,.\,\vec{b}) + (\vec{b}\,.\vec{c}) = 0$

$2\lambda - 1 = 0 \Rightarrow\ \lambda = \frac{1}{2}$

$\therefore\ \vec{r} = \frac{1}{2}\vec{a}+\vec{c}$

$\vec{r} = \frac{1}{2}(\hat{i}+\hat{j}-\hat{k}) + (\hat{i}-2\hat{j}-\hat{k})$

$\vec{r} = \frac{3}{2}\hat{i} - \frac{3}{2}\hat{j} - \frac{3}{2}\hat{k}$

$\vec{r} = \frac{3}{2}(\hat{i}-\hat{j}-\hat{k})$

$\therefore\ \vec{r}.\vec{a} = \frac{3}{2}(\hat{i}-\hat{j}-\hat{k})\,.\,(\hat{i}+\hat{j}-\hat{k}) = \frac{3}{2}(1-1+1) = \frac{3}{2}$

11. Let f and g are defined from $N \to N$ such that $f(n + 1) = f(n) + f(1)$, then which of the following is not true?

(1) f is one-one

(2) f is onto $\Rightarrow f(x) = x$

(3) if fog is one-one then g is onto

(4) if fog is onto, then g is onto

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$

$f(2) = 2f(1)$

$f(3) = 3f(1)$

$f(4) = 4f(1)$

….

$f(n) = nf(1)$

$f(x)$ is one-one

12. If the curves $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ cut each other orthogonally, then :

(1) $a - b = c - d$

(2) $ab = \frac{c+d}{a+b}$

(3) $a + b = c + d$

(4) $ab = \frac{c-d}{a-b}$

Ans. (1)

Sol. $\frac{x^2}{a}+\frac{y^2}{b}=1$ ....(1)

diff : $\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0 \Rightarrow \frac{y}{b}+\frac{dy}{dx}=\frac{-x}{a}$

$\frac{dy}{dh}=\frac{-bx}{ay}$ ....(2)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ ....(3)

diff : $\frac{dy}{dx}=\frac{-d\ x}{cy}$ ....(4)

$m_1m_2 = -1 \Rightarrow \frac{-bx}{ay}\times\frac{-dx}{cy}=-1$

$\Rightarrow bdx^2 = -acy^2$ ....(5)

$(1) - (3) \Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2+\left(\frac{1}{b}-\frac{1}{d}\right)y^2=0$

$\Rightarrow \frac{c-a}{ac}x^2+\frac{d-b}{bd}\times\left(\frac{-bd}{ac}\right)x^2=0$ (Using 5)

$\Rightarrow (c - a) - (d - b) = 0$

$\Rightarrow c - a = d - b$

$\Rightarrow c - d = a - b$

13. If : $\sin 2\theta + \tan 2\theta > 0$; $\theta \in [0, 2\pi]$. Then the complete set of values of 'θ' which satisfies

(1) $\left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{2},\frac{3\pi}{4}\right) \cup \left(\pi,\frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

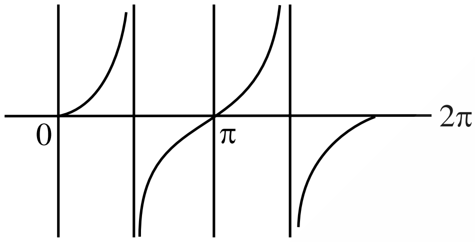
(2) $\left[0,\frac{\pi}{4}\right] \cup \left[\frac{\pi}{2},\frac{3\pi}{4}\right] \cup \left[\pi,\frac{5\pi}{4}\right] \cup \left[\frac{3\pi}{2},\frac{7\pi}{4}\right]$

(3) $\left(0,\frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4},\frac{5\pi}{4}\right)$

(4) None of these

Ans. (1)

Sol.



$\tan 2\theta\,(1 + \cos 2\theta) > 0$

$2\theta \in \left(0,\frac{\pi}{2}\right) \cup \left(\pi,\frac{3\pi}{2}\right) \cup \left(2\pi,\frac{5\pi}{2}\right) \cup \left(3\pi,\frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{2},\frac{3\pi}{4}\right) \cup \left(\pi,\frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

14. Let the quadratic equation $ax^2 + bx + c = 0$ where a,b,c are obtained by rolling the dice thrice. What is the probability that equation has equal roots.

(1) $\frac{5}{216}$ (2) $\frac{1}{72}$ (3) $\frac{1}{36}$ (4) $\frac{3}{216}$

Ans. (1)

Sol. $a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2$, $ac = 1 \Rightarrow$ $a = 1, c = 1$

If $b = 4$, $ac = 4 \Rightarrow$ $a = 1, c = 4$

$a = 4, c = 1$

$a = 2, c = 2$

If $b = 6$, $ac = 9 \Rightarrow$ $a = 3, c = 3$

$\therefore$ probability $= \frac{5}{216}$

15. Let f(x) is a polynomial of 6^{th} degree with leading co-efficient unity and $\lim\limits_{x\to 0}\dfrac{f(x)}{x^3} = 1$. Also, x = 1 & x = –1 are points of extremas of f(x), then find the value of 5f(2).

(1) 144 (2) 146 (3) 148 (4) 150

Ans. (1)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore\ f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots 1 & – 1

$\therefore\ 6 + 5a + 4b + 3 = 0\ \&\ -6 + 5a - 4b + 3 = 0$ solving

$a = -\dfrac{3}{5}\qquad b = -\dfrac{3}{2}$

$\therefore\ f(x) = x^6 - \dfrac{3}{5}x^5 - \dfrac{3}{2}x^4 + x^3$

$\therefore\ 5\,.\,f(2) = 5\left[64 - \dfrac{96}{5} - 24 + 8\right] = 144$

16. If slope at any point to a curve is $\dfrac{dy}{dx} = \dfrac{x^2 - 4x + y + 8}{(x-2)}$ and the curve passes through the origin then which of the following points also lies on the curve ?

(1) (2, 4) (2) (2, – 4) (3) (–2, –4) (4) (3, 1)

Ans. (2)

Sol. $\dfrac{dy}{dx} = \dfrac{(x-2)^2 + y + 4}{(x-2)} = (x-2) + \dfrac{y+4}{(x-2)}$

Let $x - 2 = t \Rightarrow dx = dt$

and $y + 4 = u \Rightarrow dy = du$

$\dfrac{dy}{dx} = \dfrac{du}{dt}$

$\dfrac{du}{dt} = t + \dfrac{u}{t} \Rightarrow \dfrac{du}{dt} - \dfrac{u}{t} = t$

$I.F = e^{\int \frac{-1}{t}dt} = e^{-\ln t} = \dfrac{1}{t}$

$u.\dfrac{1}{t} = \int t.\dfrac{1}{t}dt \Rightarrow \dfrac{u}{t} = t + c$

$\dfrac{y+4}{x-2} = (x-2) + c$

Passing through (0, 0)

$c = 0$

$\Rightarrow (y + 4) = (x - 2)^2$

17. Find the number of point where $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$ is non differentiable.

(1) 2 (2) 3 (3) 4 (4) 0

Ans. (2)

Sol. Non differentiable at $x = -\frac{1}{2}, -2, 1$

18. $\lim\limits_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}}{n^2}\right)^n$ is equal to

(1) $\frac{1}{e}$ (2) 1 (3) 0 (4) $\frac{1}{e^2}$

Ans. (2)

Sol. Let limit be L

So $L = e^{\lim\limits_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n}}{n}\right)} = e^k$ (say)

Now assume $n = 2^p + A, \lambda \in \{0, 1, 2, ..., 2^p - 1\}$

Now assume $1 + \left(\frac{1}{2}+\frac{1}{3}\right) + \left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right) + ... + \left(\frac{1}{2^{p-1}}+\frac{1}{2^{p-1}+1}+....+\frac{1}{2^p-1}\right)$

$+ \left(\frac{1}{2^p}+\frac{1}{2^p+1}+....+\frac{1}{2^p+\lambda}\right) = S$

So $S < 1 + \left(\frac{1}{2}+\frac{1}{2}\right) + \left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right) + ... + \underbrace{\left(\frac{1}{2^p}+\frac{1}{2^p}+....+\frac{1}{2^p}\right)}_{(\lambda+1)\text{times}}$

$\Rightarrow S < \underbrace{1+1+1......+}_{\text{p times}}\frac{\lambda+1}{2p} < p + 1$

Hence $k \le \lim\limits_{n\to\infty} \frac{p+1}{2^p} = 0$

Also $S > \underbrace{\left(\frac{1}{n}+\frac{1}{n}+\frac{1}{n}+.....+\frac{1}{n}\right)}_{\text{n times}} = 1$

Hence $k \ge \lim\limits_{n\to\infty} \frac{1}{n} = 0$

So $L = 1$

19. If the direction cosines of two lines satisfy the relations : $l + m = n$ & $l^2 + m^2 = n^2$ and 'α' is the angle between them, then the value of $\sin^4 \alpha + \cos^4 \alpha$ is :

(1) $\frac{3}{4}$ (2) $\frac{5}{8}$ (3) $\frac{1}{2}$ (4) $\frac{3}{8}$

Ans. (2)

Sol. $l^2 + m^2 + n^2 = 1$

$\therefore 2n^2 = 1 \Rightarrow n = \pm \frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 = \frac{1}{2}$ & $l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2\,l\,m = \frac{1}{2}$

$\Rightarrow l\,m = 0$ or $m = 0$

$\therefore l = 0, m = \frac{1}{\sqrt{2}}$ or $l = \frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4 \alpha + \cos^4 \alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}.\frac{3}{4} = \frac{5}{8}$

20. Let $f(x) = x^3 - ax^2 + bx + 1$ defined in [1, 2] . Rolle's theorem is applied on f(x) in [1,2] such that $f'\left(\frac{4}{3}\right) = 0$ then ordered pair (a, b) is

(1) (–5, 8) (2) (5, 8) (3) (5, –8) (4) (–5, –8)

Ans. (2)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b + 1 = 8 - 4a + 2b + 1$

$3a - b = 7$ ....(1)

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3\times\frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16$ ....(2)

$a = 5, b = 8$

21. If kx + 2y + 3z = 4, x – y – z = 5, 10x – y – 2z = 9 has infinite solutions then find the value of |k|

Ans. (11)

Sol. $D = \begin{vmatrix} k & 2 & 3 \\ 1 & -1 & -1 \\ 10 & -1 & -2 \end{vmatrix} = k(2-1) - 2(-2+10) + 3(-1+10)$

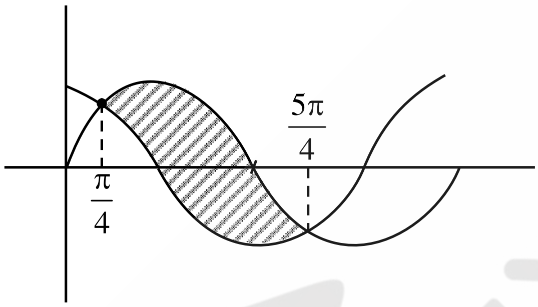
$k - 16 + 27 = 0$

$k = -11$

22. y = sinx and y = cosx intersect at many points. If area inclosed by them between two consecutive intersection points is A find A^4.

Ans. 64

Sol.



$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx = \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$\Rightarrow A^4 = (2\sqrt{2})^4 = 64$

23. If $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ and $A^2 = I$ & xyz = 2, x + y + z > 0

Find the value of $x^3 + y^3 + z^3$

Ans. 7

Sol. $A^2 = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow x^2 + y^2 + z^2 = 1$

$\Rightarrow x + y + z = 1$

$\Rightarrow xy + yz + zx = 0$

$|A|^2 = |I| \Rightarrow |A| = \pm 1 \Rightarrow 3xyz - (x^3 + y^3 + z^3) = \pm 1$

$x^3 + y^3 + z^3 = 3.2 \pm 1 = 7, 5$

$\Rightarrow x^3 + y^3 + z^3 = 7$

24. Let $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0 \ \forall \ x \in R$ then integral value of k is

Ans. 3

Sol. $D < 0$

$(2(3k - 1))^2 - 4(8k^2 - 7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k - 4)(k - 2) < 0$

$2 < k < 4$

then $k = 3$

25. The chance that a missile is intercepted is $\frac{1}{3}$. If missile is not intercepted the chance that it hits the target is $\frac{3}{4}$. the probability that all three missiles hit the target is (assume that launch of missiles are independent)

Ans. 0.125

Sol. Prob. $= \left(\frac{2}{3}.\frac{3}{4}\right)^3 = \frac{1}{8}$

26. How many numbers from 100 to 1000 using the digits 1, 2, 3, 4, 5 which are divisible by 3 or 5 (No repetition)

(1) 36 (2) 32 (3) 30 (4) 28

Ans. (2)

Sol. | | | | divisible by $\rightarrow$ 3

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

| | | 5 | = 12

4×3

Required No. = 24 + 12 – 4 = 32