Reliable INSTITUTE
Unleashing Potential

PAPER-1(B.E./B. TECH.)

JEE (Main) 2021

Questions & Solutions

(Reproduced from memory retention)

Date : 24 February, 2021 (SHIFT-1) Time ; (9.00 am to 12.00 pm)

Duration : 3 Hours | Max. Marks : 300

SUBJECT : MATHEMATICS

MATHEMATICS

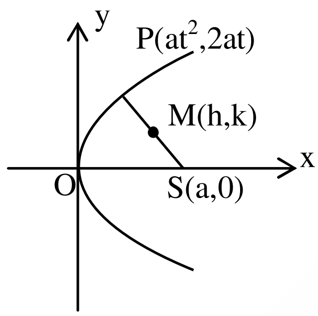
1. The locus of mid-point of the line segment joining focus of parabola $y^2 = 4ax$ to a point moving on it, is a parabola equation of whose directrix is

(1) $y = 0$ (2) $x = 0$ (3) $x = a$ (4) $y = a$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$



$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

2. There are 6 Indians 8 foreigners

Find number of committee form with atteast 2 Indians such that number of foreigners is twice the number of Indians.

(1) 1625 (2) 1050 (3) 1400 (4) 575

Ans. (1)

Sol. (2I , 4F) + (3I , 6F) + (4I , 8F)

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

3. There are two positive number p and q such that $p + q = 2$ and $p^4 + q^4 = 272$. Find the quadratic equation whose roots are p and q.

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 135 = 0$ (3) $x^2 - 2x + 16 = 0$ (4) $x^2 - 2x + 130 = 0$

Ans. (3)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

4. A fair die is thrown n times. The probability of getting an odd number twice is equal to that getting an even number thrice. The probability of getting an odd number, odd number of times is

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{8}$

Ans. (3)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$

success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

5. Population of a town at time t is given by the differential equation $\frac{dP(t)}{dt} = (0.5)P(t) - 450$. Also P(0) = 850 find the time when population of town becomes zero.

(1) $\ell n9$ (2) $3\ell n4$ (3) $2\ell n18$ (4) $\ell n18$

Ans. (3)

Sol. $\frac{dP(t)}{dt} = \frac{P(t) - 900}{2}$

$\int_0^t \frac{dP(t)}{P(t) - 900} = \int_0^t \frac{dt}{2}$

$\{\ell n\,|P(t) - 900|\}_0^t = \left\{\frac{t}{2}\right\}_0^t$

$\ell n|P(t) - 900| - \ell n|P(0) - 900| = \frac{t}{2}$

$\ell n|P(t) - 900| - \ell n50 = \frac{t}{2}$

Let at $t = t_1$, P(t) = 0 hence

$\ell n|P(t) - 900| - \ell n\,50 = \frac{t_1}{2}$

$t_1 = 2\ell n18$

6. Which of following is tautology ?

(1) $A \wedge (A \to B) \to B$
(2) $B \to (A \wedge A \to B)$
(3) $A \wedge (A \vee B)$
(4) $(A \vee B) \wedge A$

Ans. (1)

Sol. $A \wedge (\sim A \vee B) \to B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \to B$

$= (A \wedge B) \to B$

$= \sim A \vee \sim B \vee B$

$= t$

7. The value of $(-{}^{15}C_1 + 2.\,{}^{15}C_2 - 3\,{}^{15}C_3 + ...-15\,{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + ...+\,{}^{14}C_{11})$ is

(1) $2^{16} - 14$
(2) $2^{13} - 14$
(3) $2^{13} - 13$
(4) 2^{14}

Ans. (2)

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - ....-15.{}^{15}C_{15}$

$= \sum_{r=1}^{15}(-1)^r.r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$

$= 15\,(-{}^{14}C_0 + {}^{14}C_1 - .....-\,{}^{14}C_{14}) = 15\,(0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + ...+\,{}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + ...+\,{}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

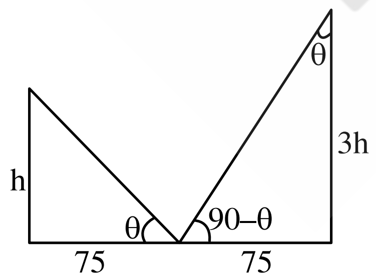
$S_1 + S_2 = 2^{13} - 14$

8. Two towers are 150m distance apart. Height of one tower is thrice the other tower. The angle of elevation of top of tower from midpoint of their feets are complement to each other then the height of smaller tower is

(1) $25\sqrt{3}$ m
(2) $\frac{25}{\sqrt{3}}$ m
(3) $75\sqrt{3}$ m
(4) 25 m

Ans. (1)

Sol.



$\tan\theta = \frac{h}{75} = \frac{75}{3h}$

$\Rightarrow\ h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}$ m

9. Tangent at point $P(t, t^3)$ of curve $y = x^3$ meets the curve again at Q then ordinate of point which divides PQ in 1 : 2 internally , is

(1) 0 (2) $2t^3$ (3) $-2t^3$ (4) 8t

Ans. (3)

Sol. equation of tangent at $P(t,t^3)$

$(y - t^3) = 3t^2 (x - t)$ ....(1)

now solve the above equation with

$y = x^3$ ....(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point = $\frac{2t^3 + (-8t^3)}{3} = -2t^3$

10. Let $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$ then f(x)

(1) decreases in $\left[\frac{1}{2}, \infty\right)$ (2) increases in $\left[\frac{1}{2}, \infty\right)$

(3) decreases in$(-\infty, \infty)$ (4) increases in $\left(-\infty, \frac{1}{2}\right]$

Ans. (2)

Sol. $f'(x) = (2x - 1)(x - \sin x)$

$\Rightarrow f'(x) \geq 0$ in $x \in \left[\frac{1}{2}, \infty\right)$

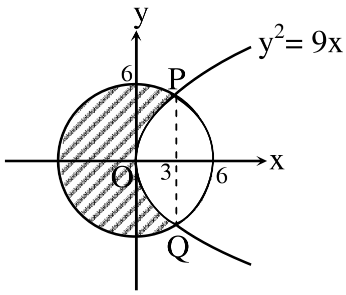
and $f'(x) \leq 0$ in $x \in \left(-\infty, \frac{1}{2}\right]$

11. The area bounded by region inside the circle $x^2 + y^2 = 36$ and outside the parabola $y^2 = 9x$ is

(1) $12\pi + 3\sqrt{3}$ (2) $36\pi + 3\sqrt{3}$ (3) $24\pi - 3\sqrt{3}$ (4) $24\pi - 3\sqrt{3}$

Ans. (3)

Sol. The curves intersect at points $(3, \pm 3\sqrt{3})$



Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36-x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left|\frac{x}{2}\sqrt{36-x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right|_3^6$$

$$= 36p - 12\sqrt{3} - 2\left(9\pi - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$

12. The equation of plane perpendicular to planes 3x + y – 2z + 1 = 0 and 2x – 5y – z + 3 = 0 such that it passes through point (1, 2, –3)

(1) 11x + y + 17z + 38 = 0 (2) 11x – y – 17z + 40 = 0

(3) 11x + y – 17z + 36 = 0 (4) x + 11y + 17z + 3 = 0

Ans. (1)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ +11 (x – 1) + (y – 2) + 17 (z + 3) = 0

11x + y + 17z + 38 = 0

13. If f : R $\rightarrow$ R is a function defined by f(x) = [x – 1] cos $\left(\frac{2x-1}{2}\right)\pi$, where[x] denotes the greatest integer function, then f is :

(1) continuous for every real x.

(2) discontinuous only at x = 1.

(3) discontinuous only at non-zero integral values of x.

(4) continuous only at x = 1.

Ans. (1)

Sol. Doubtful points are $x = n, n \in I$

$$\text{L.H.L} = \lim_{x\to n^-} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

$$\text{R.H.L.} = \lim_{x\to n^+} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2n-1}{2}\right)\pi = 0$$

$f(n) = 0$

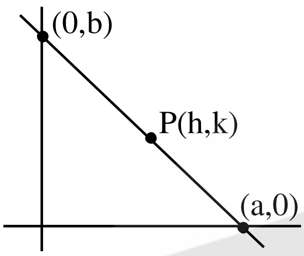
Hence continuous

14. A point is moving on the line such that the AM of reciprocal of interscepts on axis is $\frac{1}{4}$. There are 3 stones whose position are (2, 2) (4, 4) and (1, 1). Find the stone which satisfies the line

(1) (2, 2) (2) (4, 4) (3) (1, 1) (4) All of above

Ans. (1)

Sol.



$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{h}{a} + \frac{k}{b} = 1 \qquad \text{........ (i)}$$

$$\frac{\frac{1}{a}+\frac{1}{b}}{2} = \frac{1}{4}$$

$$\therefore \frac{1}{a}+\frac{1}{b}=\frac{1}{2} \qquad \text{........ (ii)}$$

$\therefore$ Line passes through fixed point (2, 2)

(from (1) and (2))

15. If $e^{(\cos^2\theta+\cos^4\theta+....\infty)\ln 2}$ is a root of equation $t^2 - 9t + 8 = 0$ then then value of $\frac{2\sin\theta}{\sin\theta+\sqrt{3}\cos\theta}$ when $0 < \theta < \frac{\pi}{2}$, is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 4

Ans. (1)

Sol. $e^{(\cos^2\theta+\cos^4\theta+....\infty)\ln 2} = 2^{\cos^2\theta+\cos^4\theta+....\infty}$

$= 2^{\cot^2\theta}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2\theta} = 1, 8 \Rightarrow \cot^2\theta = 0, 3$

$0 < \theta < \frac{\pi}{2} \Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \frac{2\sin\theta}{\sin\theta+\sqrt{3}\sin\theta} = \frac{2}{1+\sqrt{3}\cot\theta} = \frac{2}{4} = \frac{1}{2}$

16. If $I = \int \frac{\cos\theta-\sin\theta}{\sqrt{8-\sin 2\theta}}\,d\theta = a\sin^{-1}\left(\frac{\sin\theta+\cos\theta}{b}\right) + C$

then ordered pair (a, b) is

(1) (1, 3) (2) (3, 1) (3) (1, 1) (4) (–1, 3)

Ans. (1)

Sol. put $\sin\theta + \cos\theta = t \Rightarrow 1 + \sin 2\theta = t^2$

$\Rightarrow (\cos\theta - \sin\theta)\,d\theta = dt$

$\therefore I = \int \frac{dt}{\sqrt{8-(t^2-1)}} = \int \frac{dt}{\sqrt{9-t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin\theta+\cos\theta}{3}\right) + C$

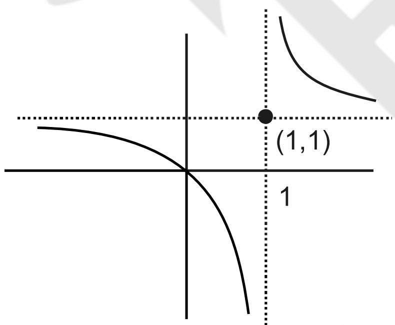
$\Rightarrow a = 1$ and $b = 3$

17. Such that $f: R \to R$, $f(x) = 2x - 1$, $g(x) = \frac{x-\frac{1}{2}}{x-1}$, then $f(g(x))$ is

(1) one-one, onto (2) many-one, onto (3) one-one, into (4) many-one, into

Ans. (3)

Sol.



$f(g(x)) = 2g(x) - 1$

$= 2\frac{\left(x-\frac{1}{2}\right)}{x-1} = \frac{x}{x-1}$

$f(g(x)) = 1 + \frac{1}{x-1}$

one-one, into

18. The distance of the point P(1,1,9) from the point of intersection of plane x + y + z = 17 and line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$

(1) $\sqrt{38}$ (2) $\sqrt{39}$ (3) 6 (4) 7

Ans. (1)

Sol. $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

$\Rightarrow x = \lambda + 3$, $y = 2\lambda + 4$, $z = 2\lambda + 5$

Which lies on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda = \frac{5}{5} = 1$

Hence , point of intersection is Q (4,6,7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

19. The value of $\lim_{x\to 0^+} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3}$

(1) $\frac{1}{15}$ (2) $\frac{2}{3}$ (3) 3 (4) 2

Ans. (2)

Sol. $\lim_{x\to 0^+} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3} = \lim_{x\to 0^+} \frac{(\sin|x|)2x}{3x^2} = \lim_{x\to 0^+} \left(\frac{\sin x}{x}\right) \times \frac{2}{3} = \frac{2}{3}$

20. The values of k and m such that system of equations 3x + 2y – k z = 10, x – 2y + 3z = 3, x + 2y – 3z = 5m are inconsistent.

(1) k = 3 and $m \neq \frac{7}{10}$ (2) k = 3 and $m = \frac{7}{10}$

(3) $k \neq 3$ and $m = \frac{7}{10}$ (4) k = 2 and $m \neq \frac{7}{10}$

Ans. (1)

Sol. $\Delta = \begin{vmatrix} 3 & 2 & -k \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow k = 3$

$$\Delta_x = \begin{vmatrix} 10 & 2 & -3 \\ 3 & -2 & 3 \\ 5m & 2 & -3 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 1 & 3 & 3 \\ 1 & 5m & -3 \end{vmatrix} = 6(7-10m)$$

$$\Delta_z = \begin{vmatrix} 3 & 2 & 10 \\ 1 & -2 & 3 \\ 1 & 2 & 5m \end{vmatrix} = 4(7-10m)$$

Hence , $k = 3$ and $m \neq \frac{7}{10}$

21. $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r^2+r}\right)\right) =$

Ans. 01.00

Sol. $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

22. Of the three independent events B_1, B_2 and B_3, the probability that only B_1 occurs is α,only B_2 occurs is β and only B_3 occurs is γ. Let the probability p that none of events B_1, B_2 or B_3 occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\frac{\text{Probability of occurrence of } B_1}{\text{Probability of occurrence of } B_3} =$

Ans. 6

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1-y)(1-z) = \alpha \qquad \Rightarrow y(1-x)(1-z) = \beta$

$\Rightarrow z(1-x)(1-y) = \gamma \qquad \Rightarrow (1-x)(1-y)(1-z) = P$

Putting in the given relation we get $x = 2y$ and $y = 3z \quad \Rightarrow \quad x = 6z \quad \Rightarrow \quad \frac{x}{z} = 6$

23. $\vec{c}$ is coplanar with $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = 2\hat{i} + \hat{k}$, $\vec{a}.\vec{c} = 7$ & $\vec{c} \perp \vec{b}$. then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is .

Ans. 75.00

Sol. $\vec{c} = \lambda\left(\vec{b} \times \left(\vec{a} \times \vec{b}\right)\right)$

$= \lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b} - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$= \lambda\left(5\left(-\hat{i} + \hat{j} + \hat{k}\right) + 2\hat{i} + \hat{k}\right)$

$= \lambda\left(-3\hat{i} + 5\hat{j} + 6\hat{k}\right)$

$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\lambda = \frac{1}{2}$

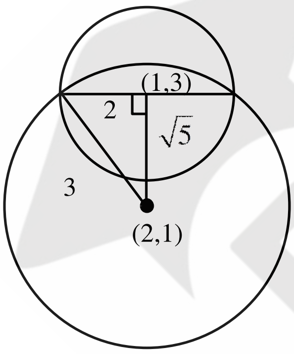
$\therefore 2\left|\left(\frac{-3}{2} - 1 + 2\right)\hat{i} + \left(\frac{5}{2} + 1\right)\hat{j} + (3 + 1 + 1)\hat{k}\right|^2$

$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right)$ = 25 + 50 = 75

24. One of the diameter of circle $C_1 : x^2 + y^2 - 2x - 6y + 6 = 0$ is chord of circle C_2 with centre (2, 1) then radius of C_2 is

Ans. **3**

Sol.



distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

25. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix such that PQ = k I, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then value of $k^2 + \alpha^2$ is equal to

Ans. 17

Sol. As $PQ = kI \Rightarrow Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(\text{adj}P)\, I \Rightarrow Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha-4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8} \Rightarrow \frac{k}{(20+12\alpha)}(-3\alpha-4) = \frac{-k}{8} \Rightarrow 2(3\alpha+4) = 5+3\alpha$

$3\alpha = -3 \Rightarrow \alpha = -1$

also $|Q| = \frac{k^3|I|}{|P|} \Rightarrow \frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$

26. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 7 ?

Ans. 540

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case- II : One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540

27. $z + \alpha|z-1| + 2i = 0$; $z \in C$ & $\alpha \in R$, then the value of $4[(\alpha_{max})^2 + (\alpha_{min})^2]$ is

Ans. 10

Sol. $x + iy + \alpha\sqrt{(x-1)^2 + y^2} + 2i = 0$

$\therefore$ $y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2+y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \frac{x^2}{x^2-2x+5}$

$\alpha^2 \in \left[0, \frac{5}{4}\right]$

$\therefore$ $\alpha^2 \in \left[0, \frac{5}{4}\right]$

$\therefore$ $\alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

then $4[(\alpha_{max})^2 + (\alpha_{min})^2] = 4\left[\frac{5}{4} + \frac{5}{4}\right] = 10$

28. Let A = {x : x is 3 digit number}

B = {x : x = 9K + 2, $k \in I$}

C : {x : x = 9K + ℓ, $k \in I$, $\ell \in I$, $0 < \ell < 9$}

If sum of elements in $A \cap (B \cup C)$ is 274×400 then ℓ is

Ans. 5.00

Sol. 3 digit number of the form 9K + 2 are {101,109,...............,992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093)

Similarly sum of 3 digit number of the form 9K + 5 is $\frac{100}{2}$ (1099)

$$\frac{100}{2}(1093) + \frac{100}{2}(1099) = 100 \times (1096)$$

$$= 400 \times 274$$

$$\Rightarrow \ell = 5$$

29. The least value of α such that $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $x \in \left(0, \frac{\pi}{2}\right)$

Ans. 9.00

Sol. Let $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$

$$y = \frac{4-3\sin x}{\sin x(1-\sin x)}$$

Let sin x = t when $t \in (0,1)$

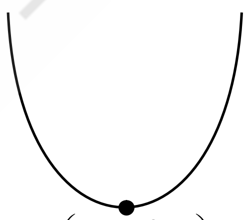
$$y = \frac{4-3t}{t-t^2}$$

$$\frac{dy}{dt} = \frac{-3(t-t^2)-(1-2t)(4-3t)}{(t-t^2)^2} = 0$$

$\Rightarrow 3t^2 - 3t - (4 - 11t + 6t^2) = 0$

$\Rightarrow 3t^2 - 8t + 4 = 0$

$\Rightarrow 3t^2 - 6t - 2t + 4 = 0$



$\left(\sin^{-1}\frac{2}{3}, 9\right)$

$\Rightarrow t = \frac{2}{3}$

$\Rightarrow \alpha \geq 9$

least α is equal to 9

30. $\int_{-a}^{a} |x| + |x-2| = 22$, a > 2 then the value of $\int_{-a}^{a} x + [x]$ is

(where [.] represent greatest integer function)

Ans. –3

Sol. $\int_{-a}^{0} (-2x+2)\ dx + \int_{0}^{2} (x+2-x)\ dx + \int_{2}^{a} (2x-2)\ dx = 22$

$x^2 - 2x\Big|_0^{-a} + 2x\Big|_0^2 + x^2 - 2x\Big|_2^a = 22$

$a^2 + 2a + 4 + a^2 - 2a - (4-4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$\int_{-3}^{3} (x+[x])\ dx = -3 - 2 - 1 + 1 + 2 = -3$